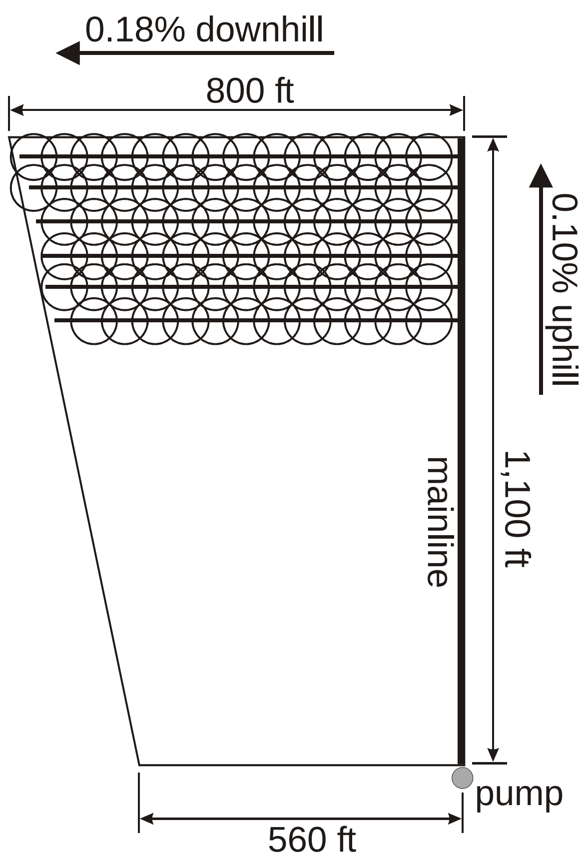
BIE 5110/6110
Sprinkle & Trickle Irrigation
Fall Semester, 2004

Assignment #7 (200 pts)
Operating Point for Fixed Sprinkler System
Due: 29 Oct 04

Given:

- A fixed sprinkler system (all sprinklers operate simultaneously) in an orchard.
- Buried IPS-PVC 1-½" lateral pipes (see Table 8.3 of the textbook).
- Rainbird® under-tree M20VH impact sprinklers with 5/64-inch SBN-1 nozzle (see www.rainbird.com for technical specifications).
- Lateral spacing: S_l = 40.00 ft.
- Sprinkler spacing: S_e = 40.00 ft.
- The field is trapezoidal in shape, as shown below:



- In the above figure, only some of the laterals and sprinklers are shown, but remember that the field area is covered with sprinklers in a fixed, permanent system.
- There are 27 laterals along the mainline.

- Note the field slopes along mainline and lateral directions.
- The suction (upstream) side of the pump has a 4-ft static lift from a pond and 10 ft of 8” PVC pipe (ID = 8.205”) with one 90-degree elbow and a strainer screen at the inlet.
- The mainline is 8” PVC pipe (ID = 8.205”), and is 1,100-ft long.
- Sprinkler riser height is: h_r = 3.0 ft.
- A Berkeley model 4GQH pump curve as shown on the following page. You will use the characteristic curve for 1600 RPM.
- Ignore minor losses along the mainline and laterals.

Required:

1. Determine the irrigated area (acres).
2. Develop an equation for sprinkler flow rate (q) as a function of pressure (P), whereby $q = K_d P^x$ (you determine K_d and x based on manufacturer’s data). Note that for a straight-bore nozzle, you would expect x to be very close to 0.5, as in Eq. 5.1a.
3. Determine the number of sprinklers along each lateral, where lateral #1 is the closest to the pump, and lateral #27 is the furthest from the pump.
4. Develop at least five points on the system curve and present those points numerically in a table. You should write a macro (Excel) or computer program to do this (don’t attempt to do it by hand with only a calculator).
5. Plot the system curve points on the attached pump curve graph, and draw a smooth curve through the points.
6. Determine the operating point (Q in gpm & TDH in ft) for this system.
7. Determine the average application rate for the whole field area.

Do you work in an organized, neat way. Make comments about assumptions and other technical issues as appropriate. Turn in all your work, including the code for your macro or computer program.

Solution:

1. The irrigated area is approximately:

$$A = \frac{0.5(800 + 560)(1100)}{43,560} = 17.2 \text{ acres}$$

   Due to the trapezoidal field shape, and the fact that there are 27 laterals at S_l = 40 ft, the effective irrigated area is slightly less than 17.2 acres.

2. A linear regression is performed on logarithms of the manufacturer's data for P and q (Rainbird® M20VH with 5/64-inch SBN-1 nozzle):

Pressure (psi)	**Flow (gpm)**	**ln(P)**	**ln(q)**
25	0.88	3.2189	-0.1278
30	0.97	3.4012	-0.0305
35	1.05	3.5553	0.0488
40	1.12	3.6889	0.1133
45	1.19	3.8067	0.1740
50	1.25	3.9120	0.2231

   given an R_2 of 0.9996, and the following equation:

$$q = 0.173P^{0.506}$$

   for q in gpm; and P in psi. Note that the exponent is close to 0.500, which is expected for a straight-bore nozzle. But notice also that allowing for the flexibility in the exponent, x, gives a better mathematical fit to the manufacturer's data.

3. Let the first lateral be located at ½S_l from the lower edge of the field (where the pump is located). Calculate the number of sprinklers per lateral by rounding the potential lateral length by S_e. The potential lateral length is calculated by linear interpolation along the left side of the field area (see the figure given above). The equation is given below, and the calculation results are shown in the following table.

$$L = 800 - (1{,}100 - y)\left(\frac{800 - 560}{1{,}100}\right)$$

   where y is the distance along the mainline (ft); and L is the potential lateral length (ft).

Lateral	Distance (ft)	Length (ft)	No. of Sprinklers
1	20	564.4	14
2	60	573.1	14
3	100	581.8	15
4	140	590.5	15
5	180	599.3	15
6	220	608.0	15
7	260	616.7	15
8	300	625.5	16
9	340	634.2	16
10	380	642.9	16
11	420	651.6	16
12	460	660.4	17
13	500	669.1	17
14	540	677.8	17
15	580	686.5	17
16	620	695.3	17
17	660	704.0	18
18	700	712.7	18
19	740	721.5	18
20	780	730.2	18
21	820	738.9	18
22	860	747.6	19
23	900	756.4	19
24	940	765.1	19
25	980	773.8	19
26	1,020	782.5	20
27	1,060	791.3	20

4. Develop a computer program to start with a given pressure at the furthest downstream sprinkler on lateral #27, calculate the flow rate at that sprinkler, calculate the pressure at the next upstream sprinkler, then the flow rate at that sprinkler, and so on, until reaching the mainline. Calculate the pressure in the mainline at the location of lateral #26, then iterate along lateral #26 to get the same pressure in the mainline at that location. Repeat for all other laterals, moving in the upstream direction, until a pressure is obtained for the upstream end of the mainline. The system flow rate is known from these calculations (sum of all individual sprinklers). Assume pipe leakage is zero.

   Knowing the system flow rate, determine the losses in the suction side of the pipe, and determine TDH by adding the velocity head at the beginning of the mainline, the pressure head at the beginning of the mainline, the static lift on the suction side, and the hydraulic losses on the suction side of the pump.

<u>Preliminary calculations and assumptions:</u>

Assume a water temperature of 10°C, giving a kinematic viscosity of: $1.306(10)^{-6}$ m^2/s, or $1.406(10)^{-5}$ ft^2/s.

Use the Swamee-Jain equation with $\varepsilon = 1.5(10)^{-6}$ m, or $4.92(10)^{-6}$ ft for PVC to obtain the Darcy-Weisbach friction factor, f.

From Table 8.3, the ID of the lateral pipe is 1.754 inches (0.1462 ft). The ID of the mainline pipe is 8.205 inches (0.6838 ft).

Once the pressure at the upstream end of the mainline is calculated, the additional TDH values include static lift, velocity head, and losses in the suction side of the pump, plus the riser height. Assume the pump outlet is at the same elevation as the upstream end of the mainline (you don't know if the mainline or laterals are buried, nor how deep they might be, but this information could be used to develop a somewhat more specific design).

$$TDH = 2.308 P_{main} + h_{lift} + h_r + \left(h_f\right)_{suction} + \frac{V^2}{2g}$$

where TDH is in ft; P_{main} is the pressure at the upstream end of the mainline (psi); h_{lift} is given as 4.0 ft; h_r is given as 3.0 ft; $(h_f)_{suction}$ are the hydraulic losses in the suction pipe (ft); and the last term is the velocity head (ft).

The iterative part is to determine P_{main}; the rest of the TDH terms are easy to calculate directly.

From Table 11.2 (minor loss coefficients):

8-inch "basket strainer": $K_r = 0.75$
8-inch "regular 90-deg elbow": $K_r = 0.26$

Velocity head in suction pipe:

$$\frac{V^2}{2g} = \frac{8Q^2}{g\pi^2 D^4} = 0.1151 Q^2$$

Friction loss in suction pipe:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 1.684 \, f \, Q^2$$

Putting it all together:

$$TDH = 2.308 P_{main} + 4.0 + 3.0 + 1.684\, f\, Q^2 + 0.1151 Q^2 \left(1 + 0.75 + 0.26\right)$$

or,

$$TDH = 2.308 P_{main} + 7.0 + Q^2 \left[1.684\, f + 0.2314\right]$$

The results are given in the table below:

P (psi)	Q_s (gpm)	P_{main} (psi)	R_e	f	TDH (ft)
20	367.2	21.8	108,347	0.01761	57.50
25	411.2	27.2	121,318	0.01721	70.09
30	451.0	32.7	133,061	0.01689	82.66
35	487.6	38.1	143,853	0.01663	95.21
40	521.6	43.5	153,902	0.01641	107.74
45	553.6	48.9	163,344	0.01622	120.26
50	583.9	54.3	172,277	0.01605	132.77
55	612.7	59.7	180,777	0.01590	145.28
60	640.2	65.1	188,901	0.01577	157.77

where P is the pressure at the furthest downstream sprinkler on lateral #27; Q_s is the total system flow rate; P_{main} is the pressure at the upstream end of the mainline (just downstream of the pump); R_e is the Reynold's number in the suction pipe; f is the value from Swamee-Jain; and TDH is the total dynamic head.

5. The system curve (Qs versus TDH) is superimposed upon the pump manufacturer's curves, as shown in the figure below.

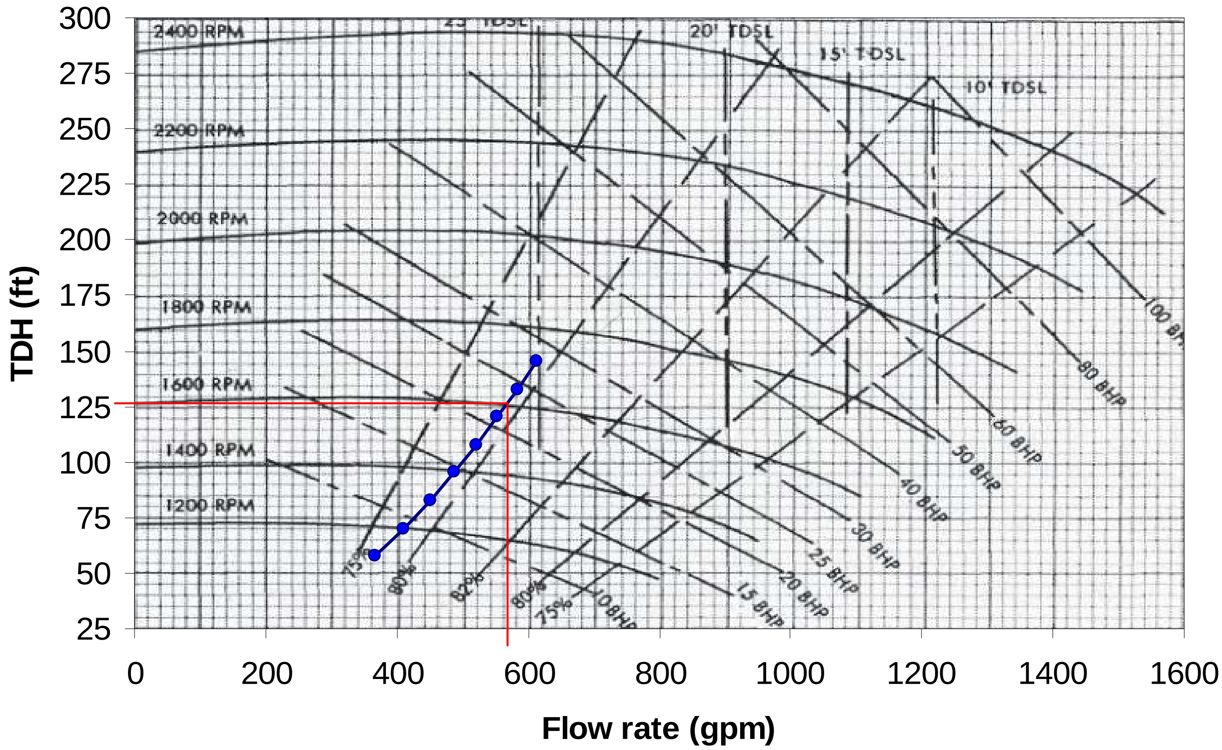
6. The operating point (intersection of the system curve and the 1600 RPM pump curve) is seen to be approximately:

$$Q_s = 568 \text{ gpm}$$
$$TDH = 126 \text{ ft}$$

7. The average application rate at this operating point is approximately:

$$AR_{avg} = \frac{(568)(12)(3600)}{(458)(40)(40)(448.86)} = 0.075 \text{ inch/hr}$$

or 1.9 mm/hr.



The MS Excel VBA listing follows:

```
Const Pi = 3.141593
Const Se = 40#           'ft
Const Sl = 40#           'ft
Const LatCount = 27      'number of laterals
Const Dlat = 0.1462      'ft
Const Dmain = 0.6838     'ft
Const SoLat = -0.0018    'ft/ft
Const SoMain = 0.001     'ft/ft

Function hf(ByVal Q As Double, ByVal D As Double, ByVal L As Double) As Double
'-------------------------------------------------------------------------------
' Returns friction loss in feet of water head (Darcy-Weisbach).
' If turbulent, uses the Swamee-Jain equation for the friction factor, f.
'-------------------------------------------------------------------------------

  Dim Re As Double, RelRough As Double, f As Double

  viscosity = 0.00001406  'ft2/s

  RelRough = 0.00000492 / D
  Re = 4 * Q / (viscosity * Pi * D)

  If Re > 4000 Then
    f = RelRough / 3.75 + 5.74 / Re ^ 0.9   'Turbulent
    f = WorksheetFunction.Log10(f)
    f = 0.25 / f ^ 2
  Else
    f = 64 / Re                             'Laminar
  End If
```

```
  hf = 8# * f * L * (Q / Pi) ^ 2 / (32.2 * D ^ 5)

End Function

Function Lateral(ByVal P As Double, n As Integer, Qlat As Double) As Double
'------------------------------------------------------------------------------
' Calculates lateral inlet pressure head for "n" sprinklers.
' Pressure, P, is in psi.
'------------------------------------------------------------------------------

  Dim Qs As Double, h As Double

  Qlat = 0# 'cfs

  For i = 1 To n
    Qs = 0.173 * P ^ 0.506                                'gpm
    Qlat = Qlat + Qs / 448.86                             'cfs
    h = P * 2.308 + hf(Qlat, Dlat, Se) + SoLat * Se       'ft
    P = h / 2.308                                         'psi
  Next

  Lateral = h

End Function

Function ParabolaFit(x, y, target As Double) As Double
'------------------------------------------------------------------------------
' Determines constants a, b, and c for a parabola through three points.
' Equation is: y = ax^2 + bx + c.  If parabola impossible, uses bisection.
' Returns the x-value (P) which matches the specified target y-value (hmain).
' P is the pressure at the furthest DS sprinkler in the lateral.
'------------------------------------------------------------------------------

  Dim Bisection As Boolean
  Dim foo As Double, boo As Double
  Dim a As Double, b As Double, c As Double

  ParabolaFit = 0
  foo = x(2) - x(1)
  Bisection = Abs(foo) < 0.0000000001

  If Not Bisection Then

    C0 = x(1) ^ 2
    c1 = (x(3) - x(1)) / foo
    c2 = x(2) ^ 2 - C0

    boo = x(3) ^ 2 - C0 - c1 * c2
    Bisection = Abs(boo) < 0.0000000001

    If Not Bisection Then

      '-------------------------
      ' Parabolic interpolation
      '-------------------------

      a = ((y(1) - y(2)) * c1 - y(1) + y(3)) / boo
      b = (y(2) - y(1) - a * c2) / foo
      c = y(2) - x(2) * (a * x(2) + b)

      c = c - target
      ParabolaFit = (-b + Sqr(b * b - 4 * a * c)) / (2 * a)
      Exit Function
    End If
  End If
```

```
  If Bisection Then

    '--------------
    ' Use bisection
    '--------------

    If y(2) < target Then
      ParabolaFit = (y(2) + y(3)) / 2
    Else
      ParabolaFit = (y(2) + y(1)) / 2
    End If
  End If

End Function

Function QsPmain(ByVal P As Double, Flow As Boolean) As Double
'------------------------------------------------------------------------------
' Iterates to determine the system flow rate for a given starting pressure.
' Returns either flow rate (Qs) or pressure (Pmain) at US end of the mainline.
'------------------------------------------------------------------------------

  Dim n As Integer
  Dim lat As Integer
  Dim LatHead(1 To 3) As Double
  Dim Pressure(1 To 3) As Double
  Dim Qlat As Double, x As Double
  Dim Qmain As Double, L As Double, NewP As Double, hmain As Double

  hmain = 0
  Qmain = 0

  For lat = LatCount To 1 Step -1

    '-------------------------------------------
    ' Determine number of sprinklers this lateral
    '-------------------------------------------

    x = 20 + (lat - 1) * Sl
    L = 800# - (1100# - x) * 0.2181818
    n = Round(L / Se, 0)

    '-------------------------------------
    ' Calculate lateral inlet pressure head
    '-------------------------------------

    If lat = LatCount Then

      '-----------------------------------
      ' No need to iterate for last lateral
      '-----------------------------------

      LatHead(2) = Lateral(P, n, Qlat)
    Else

      '------------------------------------------------
      ' Iterate to match lateral inlet & mainline heads
      '------------------------------------------------

      Pressure(1) = P / 4
      Pressure(3) = 3 * P
      Pressure(2) = (Pressure(1) + Pressure(3)) / 2

      LatHead(1) = Lateral(Pressure(1), n, Qlat)
      LatHead(3) = Lateral(Pressure(3), n, Qlat)
      LatHead(2) = Lateral(Pressure(2), n, Qlat)
```

```
      If (LatHead(1) > hmain) Or (LatHead(3) < hmain) Then

        '-------------------------------
        ' Failed to bracket the solution
        '-------------------------------

        QsPmain = -100
        Exit Function
      End If

      For i = 1 To 50

        '---------------------------------
        ' Search by parabolic interpolation
        '---------------------------------

        NewP = ParabolaFit(Pressure, LatHead, hmain)

        If NewP < Pressure(2) Then
          Pressure(3) = Pressure(2)
          LatHead(3) = LatHead(2)
        Else
          Pressure(1) = Pressure(2)
          LatHead(1) = LatHead(2)
        End If

        Pressure(2) = NewP
        LatHead(2) = Lateral(NewP, n, Qlat)

        If Abs(LatHead(2) - hmain) < 0.001 Then

          '-------------------
          ' Solution converged
          '-------------------

          Exit For
        End If
      Next

    End If

    '----------------------------------------
    ' Move upstream one hydrant along mainline
    '----------------------------------------

    Qmain = Qmain + Qlat
    hmain = LatHead(2) + hf(Qmain, Dmain, Sl) + SoMain * Sl
  Next

  '---------------------------
  ' Return the system flow rate
  ' or the mainline pressure
  '---------------------------

  If Flow Then
    QsPmain = Qmain * 448.86  'gpm
  Else
    QsPmain = hmain / 2.308   'psi
  End If

End Function
```