BIE 5300/6300 Assignment #7
Earthen Canal Design

28 Oct 04 (due 2 Nov 04)

Show your calculations in an organized and neat format, including all relevant calculations. Indicate any assumptions or relevant comments.

Given:

- An earthen canal is to be designed
- Accommodating the natural terrain, the longitudinal bed slope will be $S_o = 0.0001$
- Bed material is a non-cohesive fine sandy soil material with an average particle diameter of 2 mm
- The angle of repose for wet material is 26 degrees
- Assume a Manning n of 0.019
- Design discharge is $Q_{max} = 12\ m^3/s$
- The source water has a low content of fine sediment (silt)
- The b/h ratio should be limited to values between 2 and 6
- The inverse side slope should be limited as follows: $m < 3.5$

Required:

- Design the earthen channel section by applying the tractive force method
- Compare your results with those using the assumption of a very wide channel, in which the critical tractive force is $\gamma h S_o$
- Compare your design with the velocity as obtained from the Kennedy formula
- Compare your design with the velocity as obtained from the Lacey method
- Compare your design with the velocity as obtained from the maximum velocity method, both for values by Fortier and Scobey, and by the USBR

A Design Solution:

<u>Critical Tractive Force</u>

- The critical tractive force is taken from Fig. 5 (non-cohesive material) of the lecture notes, using the curve labeled "low content of fine sediment."
- Instead of reading the graph by eye, use the appropriate equation from the lecture notes:

$$T_c = 0.0756(2)^3 - 0.241(2)^2 + 0.872(2) + 2.26$$

$$T_c \approx 3.64\ N/m^2$$

Angle of Repose

- The angle of repose, θ, is given as 26°
- Then, the ratio of T_{side} to T_{bed} is:

$$K = \frac{T_{side}}{T_{bed}} = \sqrt{1 - \frac{\sin^2\phi}{\sin^2\theta}} = \sqrt{1 - 5.20\sin^2\phi}$$

- Design requirements for this example call for a side slope between 0.0 & 3.5
- Then,

$$\phi_{min} = \tan^{-1}\left(\frac{1}{3.5}\right) = 15.9^\circ$$

- Let ϕ_{min} = 16° (round to nearest whole degree)
- θ = 26° is the upper limit for ϕ, so: 16° ≤ ϕ ≤ 26°

- Make a table of tractive force ratio, K, values for the acceptable range of ϕ:

ϕ (deg)	**m**	**K**
16	3.487	*0.778*
17	3.271	*0.745*
18	3.078	*0.710*
19	2.904	*0.670*
20	2.747	*0.626*
21	2.605	*0.576*
22	2.475	*0.520*
23	2.356	*0.454*
24	2.246	*0.374*
25	2.145	*0.267*
26	2.050	*0.027*

Maximum Shear Stress Fractions

- These are K_{bed} and K_{side}
- The range of inverse side slopes is: 3.487 ≤ m ≤ 2.050
- Recall that the range of bed width to depth is: 2.0 ≤ b/h ≤ 6.0
- For the bed, apply the equations from the lecture notes, where K_{bed} is a function of the ratio b/h for trapezoidal cross sections…

b/h	**K_{bed}**
2.0	*0.881*
2.4	*0.906*
2.8	*0.927*
3.2	*0.946*
3.6	*0.963*
4.0	*0.969*
4.4	*0.971*
4.8	*0.973*
5.2	*0.975*
5.6	*0.977*
6.0	*0.980*

- For the sides, apply the equations from the lecture notes, where K_{side} is a function of inverse side slope, m, and the ratio b/h

		b/h										
ϕ (deg)	**m**	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
16	3.487	*0.794*	*0.793*	*0.791*	*0.789*	*0.788*	*0.787*	*0.786*	*0.785*	*0.784*	*0.784*	*0.783*
17	3.271	*0.805*	*0.806*	*0.807*	*0.807*	*0.808*	*0.809*	*0.809*	*0.810*	*0.810*	*0.811*	*0.811*
18	3.078	*0.808*	*0.812*	*0.815*	*0.818*	*0.820*	*0.822*	*0.824*	*0.825*	*0.827*	*0.828*	*0.830*
19	2.904	*0.807*	*0.812*	*0.816*	*0.820*	*0.823*	*0.826*	*0.828*	*0.831*	*0.833*	*0.835*	*0.837*
20	2.747	*0.800*	*0.806*	*0.811*	*0.815*	*0.819*	*0.822*	*0.824*	*0.827*	*0.829*	*0.831*	*0.833*
21	2.605	*0.791*	*0.797*	*0.802*	*0.806*	*0.810*	*0.812*	*0.815*	*0.817*	*0.819*	*0.821*	*0.822*
22	2.475	*0.782*	*0.787*	*0.792*	*0.796*	*0.799*	*0.801*	*0.803*	*0.805*	*0.807*	*0.808*	*0.809*
23	2.356	*0.772*	*0.777*	*0.782*	*0.785*	*0.788*	*0.790*	*0.792*	*0.793*	*0.795*	*0.796*	*0.797*
24	2.246	*0.763*	*0.768*	*0.772*	*0.775*	*0.778*	*0.780*	*0.781*	*0.783*	*0.784*	*0.785*	*0.786*
25	2.145	*0.755*	*0.760*	*0.764*	*0.767*	*0.770*	*0.771*	*0.773*	*0.774*	*0.775*	*0.776*	*0.777*
26	2.050	*0.748*	*0.754*	*0.757*	*0.760*	*0.762*	*0.764*	*0.766*	*0.767*	*0.768*	*0.768*	*0.769*

Note: values in italics are K_{side}

Maximum Allowable Water Depths

- These are h_{max} based on: (1) bed; and, (2) side slopes
- For water, use γ = 9,810 N/m^3
- The following table has h_{max} values based on K_{bed}:

	b/h										
m	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
3.487	*3.28*	*3.19*	*3.11*	*3.05*	*3.00*	*2.98*	*2.97*	*2.97*	*2.96*	*2.95*	*2.95*
3.271	*3.14*	*3.05*	*2.98*	*2.92*	*2.87*	*2.85*	*2.85*	*2.84*	*2.84*	*2.83*	*2.82*
3.078	*2.99*	*2.91*	*2.84*	*2.78*	*2.73*	*2.72*	*2.71*	*2.71*	*2.70*	*2.69*	*2.69*
2.904	*2.82*	*2.75*	*2.68*	*2.63*	*2.58*	*2.57*	*2.56*	*2.55*	*2.55*	*2.54*	*2.54*
2.747	*2.64*	*2.56*	*2.50*	*2.45*	*2.41*	*2.40*	*2.39*	*2.39*	*2.38*	*2.38*	*2.37*
2.605	*2.43*	*2.36*	*2.31*	*2.26*	*2.22*	*2.21*	*2.20*	*2.20*	*2.19*	*2.19*	*2.18*
2.475	*2.19*	*2.13*	*2.08*	*2.04*	*2.00*	*1.99*	*1.99*	*1.98*	*1.98*	*1.97*	*1.97*
2.356	*1.91*	*1.86*	*1.82*	*1.78*	*1.75*	*1.74*	*1.74*	*1.73*	*1.73*	*1.72*	*1.72*
2.246	*1.58*	*1.53*	*1.50*	*1.47*	*1.44*	*1.43*	*1.43*	*1.43*	*1.42*	*1.42*	*1.42*
2.145	*1.12*	*1.09*	*1.07*	*1.05*	*1.03*	*1.02*	*1.02*	*1.02*	*1.02*	*1.01*	*1.01*
2.050	*0.11*	*0.11*	*0.11*	*0.11*	*0.10*	*0.10*	*0.10*	*0.10*	*0.10*	*0.10*	*0.10*

Note: values in italics are h_{max} *based on* K_{bed}

- The next table has h_{max} values based on K_{side}:

	b/h										
m	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
3.487	*3.63*	*3.64*	*3.65*	*3.66*	*3.66*	*3.67*	*3.67*	*3.68*	*3.68*	*3.68*	*3.69*
3.271	*3.44*	*3.43*	*3.43*	*3.43*	*3.42*	*3.42*	*3.42*	*3.42*	*3.41*	*3.41*	*3.41*
3.078	*3.26*	*3.24*	*3.23*	*3.22*	*3.21*	*3.20*	*3.20*	*3.19*	*3.18*	*3.18*	*3.17*
2.904	*3.08*	*3.06*	*3.05*	*3.03*	*3.02*	*3.01*	*3.00*	*2.99*	*2.98*	*2.98*	*2.97*
2.747	*2.90*	*2.88*	*2.86*	*2.85*	*2.84*	*2.83*	*2.82*	*2.81*	*2.80*	*2.79*	*2.79*
2.605	*2.70*	*2.68*	*2.67*	*2.65*	*2.64*	*2.63*	*2.62*	*2.62*	*2.61*	*2.61*	*2.60*
2.475	*2.47*	*2.45*	*2.44*	*2.42*	*2.42*	*2.41*	*2.40*	*2.40*	*2.39*	*2.39*	*2.38*
2.356	*2.18*	*2.17*	*2.15*	*2.15*	*2.14*	*2.13*	*2.13*	*2.12*	*2.12*	*2.12*	*2.11*
2.246	*1.82*	*1.81*	*1.80*	*1.79*	*1.78*	*1.78*	*1.77*	*1.77*	*1.77*	*1.77*	*1.77*
2.145	*1.31*	*1.30*	*1.30*	*1.29*	*1.29*	*1.28*	*1.28*	*1.28*	*1.28*	*1.28*	*1.28*
2.050	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*	*0.13*

Note: values in italics are h_{max} based on K_{side}

- The above two tables show that h_{max} from K_{side} is greater for every combination of b/h and m (within the given ranges)
- Therefore, use only the table for h_{max} from K_{side} to determine the maximum allowable depth of water
- Note that the full range of “m” and the full range of “b/h” is represented in the above two tables

Channel Base Width Limits

- Calculate the uniform flow (normal) depth for values of “m” from 2.050 to 3.487 (as shown in the above tables), and various values of base width, “b.”

	base width, b (m)										
m	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
3.487	*1.93*	*1.88*	*1.83*	*1.79*	*1.74*	*1.70*	*1.66*	*1.62*	*1.58*	*1.55*	*1.51*
3.271	*1.96*	*1.91*	*1.86*	*1.81*	*1.76*	*1.72*	*1.68*	*1.64*	*1.60*	*1.56*	*1.53*
3.078	*2.00*	*1.94*	*1.89*	*1.83*	*1.79*	*1.74*	*1.70*	*1.65*	*1.62*	*1.58*	*1.54*
2.904	*2.02*	*1.97*	*1.91*	*1.86*	*1.81*	*1.76*	*1.71*	*1.67*	*1.63*	*1.59*	*1.55*
2.747	*2.05*	*1.99*	*1.93*	*1.88*	*1.83*	*1.78*	*1.73*	*1.69*	*1.64*	*1.60*	*1.56*
2.605	*2.08*	*2.02*	*1.96*	*1.90*	*1.84*	*1.79*	*1.75*	*1.70*	*1.66*	*1.61*	*1.58*
2.475	*2.11*	*2.04*	*1.98*	*1.92*	*1.86*	*1.81*	*1.76*	*1.71*	*1.67*	*1.63*	*1.59*
2.356	*2.13*	*2.06*	*2.00*	*1.94*	*1.88*	*1.83*	*1.77*	*1.73*	*1.68*	*1.64*	*1.60*
2.246	*2.16*	*2.09*	*2.02*	*1.96*	*1.90*	*1.84*	*1.79*	*1.74*	*1.69*	*1.65*	*1.60*
2.145	*2.18*	*2.11*	*2.04*	*1.97*	*1.91*	*1.85*	*1.80*	*1.75*	*1.70*	*1.66*	*1.61*
2.050	*2.21*	*2.13*	*2.06*	*1.99*	*1.93*	*1.87*	*1.81*	*1.76*	*1.71*	*1.66*	*1.62*

Note: values in italics are depths, h, based on the Manning equation

- Next, divide the base width by each respective depth, over the range of inverse side slope values, giving a table of b/h ratios:

	base width, b (m)										
m	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
3.487	2.07	2.39	2.73	3.08	3.44	3.82	4.22	4.63	5.05	5.49	5.94
3.271	2.04	2.35	2.69	3.04	3.40	3.78	4.17	4.58	5.00	5.44	5.89
3.078	2.00	2.32	2.65	3.00	3.36	3.73	4.13	4.53	4.95	5.39	5.84
2.904	1.98	2.29	2.62	2.96	3.32	3.70	4.08	4.49	4.91	5.34	5.79
2.747	1.95	2.26	2.59	2.93	3.29	3.66	4.05	4.45	4.87	5.30	5.75
2.605	1.92	2.23	2.56	2.90	3.25	3.62	4.01	4.41	4.83	5.26	5.71
2.475	1.90	2.21	2.53	2.87	3.22	3.59	3.98	4.38	4.80	5.23	5.68
2.356	1.88	2.18	2.50	2.84	3.19	3.56	3.95	4.35	4.76	5.20	5.64
2.246	1.85	2.16	2.48	2.81	3.16	3.53	3.92	4.32	4.73	5.16	5.61
2.145	1.83	2.14	2.45	2.79	3.14	3.51	3.89	4.29	4.70	5.13	5.58
2.050	1.81	2.11	2.43	2.76	3.11	3.48	3.86	4.26	4.68	5.11	5.55

Note: values in italics are ratios of b/h, using h from the Manning equation

- These calculations show that the range of base widths is limited (approximately) as follows (with b in meters):

$$4 < b < 9$$

whereby b < 4 gives b/h less than the minimum of 2, and b > 9 gives b/h greater than the maximum of 6, as specified for this problem

<u>Ratios of h to h_{max}</u>

- Divide depths, h, from Manning by h_{max} based on K_{side}
- Only values less than unity are acceptable for this ratio

	base width, b (m)										
m	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
3.487	0.53	0.52	0.50	0.49	0.48	0.46	0.45	0.44	0.43	0.42	0.41
3.271	0.57	0.56	0.54	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.45
3.078	0.61	0.60	0.58	0.57	0.56	0.54	0.53	0.52	0.51	0.50	0.49
2.904	0.66	0.64	0.63	0.61	0.60	0.58	0.57	0.56	0.54	0.53	0.52
2.747	0.71	0.69	0.67	0.66	0.64	0.63	0.61	0.60	0.59	0.57	0.56
2.605	0.77	0.75	0.73	0.71	0.70	0.68	0.66	0.65	0.63	0.62	0.60
2.475	0.85	0.83	0.81	0.79	0.77	0.75	0.73	0.71	0.70	0.68	0.66
2.356	0.97	0.95	0.92	0.90	0.88	0.85	0.83	0.81	0.79	0.77	0.75
2.246	**1.18**	**1.15**	**1.12**	**1.09**	**1.06**	**1.03**	**1.00**	0.98	0.95	0.93	0.91
2.145	**1.66**	**1.61**	**1.57**	**1.52**	**1.48**	**1.44**	**1.40**	**1.36**	**1.33**	**1.30**	**1.26**
2.050	**16.51**	**16.04**	**15.58**	**15.13**	**14.71**	**14.30**	**13.90**	**13.53**	**13.17**	**12.84**	**12.52**

Note: values in bold are ratios of h/h_{max} that exceed unity, and are therefore unacceptable

- The above table shows that acceptable inverse side slopes for this design problem are (rounding to two significant digits):

$$2.3 < m < 3.5$$

- The above table also shows that the base width can be any value between 4 and 9 m
- These ranges of base width and inverse side slope represent the domain of feasible design solutions for this channel
- It would often be best to limit the width of the channel, possibly choosing a value of b = 4, and accepting a somewhat greater depth of water for uniform flow

Very Wide Channel

- With this assumption, $T_c = \gamma h S_o$
- Then,

$$h_{max} = \frac{T_c}{\gamma S_o} = \frac{3.64}{9{,}810(0.0001)} = 3.71\,m$$

- The largest h_{max} value based on K_{side}, for the acceptable range of m and b, is 3.69 m (see the above table)
- Therefore, the "very wide channel" solution is less restrictive than the previous solution and will not have any bearing on the range of feasible m and b values

Kennedy Formula

- For a "fine sandy soil," C_1 = 0.84
- For water containing "fine silt," C_2 = 0.64
- Applying the Kennedy formula:

$$V_o = 0.84(0.3048)\left(\frac{h}{0.3048}\right)^{0.64}$$

where V_o is the "regime" velocity (m^3/s); and h is depth (m) from the Manning equation

- Multiply V_o by area, A, to obtain flow rate according to the Kennedy formula, where:

$$A = h(b + mh)$$

for a trapezoidal channel section

- The following table gives Q values (m^3/s) based on depths from the Manning equation and V_o from the Kennedy formula for the previously-established range of acceptable b and m values:

	base width, b (m)										
m	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
3.487	*17.3*	*17.1*	*16.9*	*16.6*	*16.4*	*16.2*	*16.1*	*15.9*	*15.7*	*15.6*	*15.5*
3.271	*17.3*	*17.0*	*16.8*	*16.6*	*16.4*	*16.2*	*16.0*	*15.8*	*15.7*	*15.5*	*15.4*
3.078	*17.2*	*17.0*	*16.7*	*16.5*	*16.3*	*16.1*	*15.9*	*15.8*	*15.6*	*15.4*	*15.3*
2.904	*17.2*	*16.9*	*16.7*	*16.5*	*16.3*	*16.1*	*15.9*	*15.7*	*15.5*	*15.4*	*15.2*
2.747	*17.2*	*16.9*	*16.7*	*16.4*	*16.2*	*16.0*	*15.8*	*15.6*	*15.5*	*15.3*	*15.2*
2.605	*17.1*	*16.9*	*16.6*	*16.4*	*16.2*	*15.9*	*15.8*	*15.6*	*15.4*	*15.3*	*15.1*
2.475	*17.1*	*16.8*	*16.6*	*16.3*	*16.1*	*15.9*	*15.7*	*15.5*	*15.4*	*15.2*	*15.1*
2.356	*17.1*	*16.8*	*16.6*	*16.3*	*16.1*	*15.9*	*15.7*	*15.5*	*15.3*	*15.2*	*15.0*
2.246	*17.1*	*16.8*	*16.5*	*16.3*	*16.0*	*15.8*	*15.6*	*15.4*	*15.3*	*15.1*	*15.0*
2.145	*17.1*	*16.8*	*16.5*	*16.2*	*16.0*	*15.8*	*15.6*	*15.4*	*15.2*	*15.1*	*14.9*
2.050	*17.1*	*16.8*	*16.5*	*16.2*	*16.0*	*15.8*	*15.6*	*15.4*	*15.2*	*15.1*	*14.9*

Note: values in italics are flow rate (m^3/s) based on V_o from Kennedy ($Q = V_oA$)

- Note that all flow rates in the above table are greater than Q_{max} (12 m^3/s)
- This means that channel scouring would not be expected, at the design discharge, according to the Kennedy formula
- However, some sediment deposition might occur, assuming the Kennedy formula is correct for these site-specific conditions
- Recall that the Kennedy formula is 100% empirical

Lacey Method

- The average particle diameter is given as d_m = 2 mm
- Then, the "f" value from Lacey is:

$$f = 1.76\sqrt{2} = 2.49$$

- Longitudinal bed slope for "regime" flow:

$$S = 0.000547\left(\frac{f^{2/3}}{Q^{1/6}}\right) = 0.000547\left(\frac{2.49^{2/3}}{(12*35.31)^{1/6}}\right) = 0.00037$$

- This is greater than the specified design bed slope of 0.0001
- Thus, Lacey's method would predict some scouring at the design discharge
- Note that other relationships from the Lacey method could also be checked
- However, recall that the Lacey method is 100% empirical

Maximum Velocity Method

- Refer to the tables from the lecture notes
- From Fortier & Scobey, use the column for "water with colloidal silt": the value for fine sand is V_{max} = 2.5 fps (0.76 m/s)

- The following table gives average velocity (m/s) at a channel cross section for uniform flow conditions, where V = Q/A, and Q = 12 m^3/s:

	base width, b (m)										
m	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
3.487	*0.58*	*0.58*	*0.57*	*0.57*	*0.57*	*0.57*	*0.57*	*0.56*	*0.56*	*0.56*	*0.55*
3.271	*0.59*	*0.58*	*0.58*	*0.58*	*0.58*	*0.58*	*0.57*	*0.57*	*0.57*	*0.56*	*0.56*
3.078	*0.59*	*0.59*	*0.59*	*0.59*	*0.58*	*0.58*	*0.58*	*0.58*	*0.57*	*0.57*	*0.57*
2.904	*0.60*	*0.60*	*0.60*	*0.59*	*0.59*	*0.59*	*0.58*	*0.58*	*0.58*	*0.58*	*0.57*
2.747	*0.61*	*0.60*	*0.60*	*0.60*	*0.60*	*0.59*	*0.59*	*0.59*	*0.58*	*0.58*	*0.58*
2.605	*0.61*	*0.61*	*0.61*	*0.60*	*0.60*	*0.60*	*0.60*	*0.59*	*0.59*	*0.58*	*0.58*
2.475	*0.62*	*0.62*	*0.61*	*0.61*	*0.61*	*0.60*	*0.60*	*0.60*	*0.59*	*0.59*	*0.59*
2.356	*0.62*	*0.62*	*0.62*	*0.62*	*0.61*	*0.61*	*0.61*	*0.60*	*0.60*	*0.59*	*0.59*
2.246	*0.63*	*0.63*	*0.62*	*0.62*	*0.62*	*0.61*	*0.61*	*0.61*	*0.60*	*0.60*	*0.59*
2.145	*0.63*	*0.63*	*0.63*	*0.62*	*0.62*	*0.62*	*0.61*	*0.61*	*0.61*	*0.60*	*0.60*
2.050	*0.64*	*0.64*	*0.63*	*0.63*	*0.63*	*0.62*	*0.62*	*0.61*	*0.61*	*0.60*	*0.60*

Note: values in italics are average velocity (m/s)

- In this case, all average velocity values within the feasible range (tractive force) of m and b are below 0.76 m/s
- Thus, no scouring is to be expected, although perhaps some sediment deposition might occur, according to Fortier & Scobey
- Again, the maximum velocity method is 100% empirical and will not provide accurate results in all cases
- From the USBR, an average particle diameter of 2 mm falls under the “coarse sand” category, giving a maximum velocity of V_{max} = 1.8 fps (0.55 m/s)
- This is more restrictive than the V_{max} from Fortier & Scobey, and also exceeds almost all of the velocity values from the above table, but not by very much