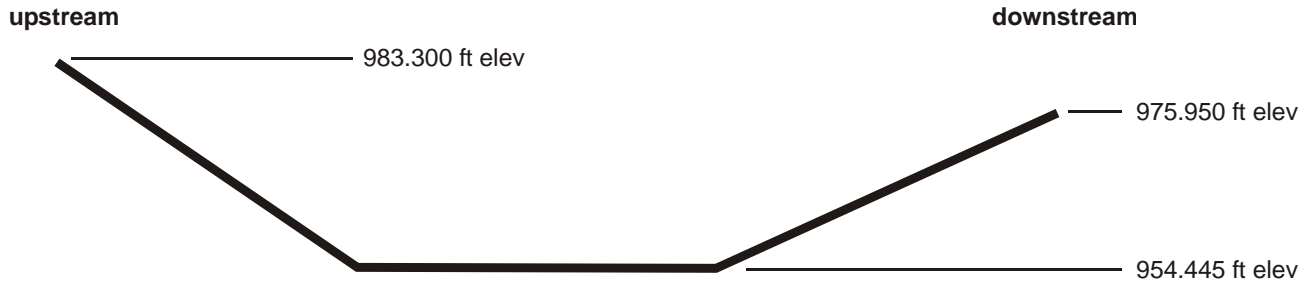


BIE 6300
 Fall Semester 2004
 Exam #2 – 14 Dec 04

Show your work neatly on this or separate pages. Show units for all calculated values. Make note of any important assumptions.

Name _____

An inverted siphon with a single barrel of circular concrete pipe (2.25-ft inside diameter) has a total length of 635 ft. The descending part of the barrel has a length of 185 ft. The inlet and outlet structures are ungated Type 1 USBR transitions. See the side view (profile) figure below:



The inverted siphon connects upstream and downstream open canals which have the same trapezoidal in cross section: base width is $b = 4.0$ ft; inverse side slope is 1.00; and, longitudinal bed slope is 0.000212 ft/ft. The Manning roughness is estimated to be $n = 0.015$ in the open canals. Assume uniform-flow conditions in the open channels both upstream and downstream of the inverted siphon.

- a) Estimate the flow rate through the inverted siphon pipe under steady-state flow conditions. Use the Darcy-Weisbach equation with friction factor “ f ” defined by the Swamee-Jain equation:

$$f = \frac{0.25}{\left[\log_{10} \left(\frac{\varepsilon}{3.75D} + \frac{5.74}{N_R^{0.9}} \right) \right]^2}$$

where the roughness height is $\varepsilon = 0.001$ ft; and the kinematic viscosity of the water is $\nu = 1.2(10)^{-5}$ ft²/s.

- b) Calculate the maximum pressure in the inverted siphon pipe for steady-state flowing conditions.
 c) Calculate the maximum pressure in the inverted siphon pipe for non-flowing conditions.
 d) Calculate the uniform-flow depth in the upstream open canal.
 e) Is the hydraulic seal sufficient at the inverted siphon entrance? (the barrel invert has the same elevation as the canal invert at the siphon entrance).

Solution:

(a) Steady-State Flow Rate

- Recognize that the open canals US and DS of the inverted siphon have the same cross-section, longitudinal bed slope, and roughness. Thus, if uniform-flow conditions prevail US and DS of the inverted siphon, the depth just US of the siphon must be the same as the depth just DS. This means that the head differential on the siphon is equal to the difference in entrance and exit elevations:

$$\Delta\text{Elev} = 983.300 - 975.95 = 7.350 \text{ ft}$$

- Under steady-state flow conditions, the velocity heads are the same in the open channels just US and just DS of the inverted siphon. This means that the hydraulic energy loss through the siphon under these conditions is very nearly equal to ΔElev . Then,

$$\Delta\text{Elev} = h_f$$

where h_f is determined from Darcy-Weisbach and Swamee-Jain (English units):

$$h_f = f \frac{L V^2}{D 2g} = \frac{8fLQ^2}{g\pi^2 D^5} = \frac{8f(635)Q^2}{(32.2)\pi^2 (2.25)^5} = 0.277fQ^2$$

- The friction factor, f , is defined as:

$$f = \frac{0.25}{\left[\log_{10} \left(\frac{0.001}{3.75(2.25)} + \frac{5.74}{N_R^{0.9}} \right) \right]^2} = \frac{0.25}{\left[\log_{10} \left(0.000119 + \frac{5.74}{N_R^{0.9}} \right) \right]^2}$$

- The Reynolds number is:

$$N_R = \frac{VD}{\nu} = \frac{4Q}{\nu\pi D} = \frac{4Q}{1.2(10)^{-5}\pi(2.25)} = 4.72(10)^4 Q$$

- Then:

$$f = \frac{0.25}{\left[\log_{10} \left(0.000119 + 0.000357 Q^{-0.9} \right) \right]^2}$$

- Combine the above equation with the equation for h_f :

$$h_f = \frac{0.0693Q^2}{\left[\log_{10} \left(0.000119 + 0.000357 Q^{-0.9} \right) \right]^2} = 7.350 \text{ ft}$$

where the only unknown is Q (cfs). Rearrange as follows:

$$Q = -10.3 \log_{10} (0.000119 + 0.000357 Q^{-0.9})$$

- Using the above equation, make an initial guess of 10 cfs (for example), then iterate to determine the solution to the equation:

Q (cfs)	next Q (cfs)
10.00	38.99
38.99	39.95
39.95	39.96

- From the above table, $Q = 40.0$ cfs (to three significant digits). This gives $f = 0.0166$.

(d) Uniform Flow Depth in the Canal

- Use the Manning equation:

$$40.0 = \frac{1.49}{0.015} \frac{[h(4.0 + h)]^{5/3}}{[4.0 + 2h\sqrt{1+1}]^{2/3}} \sqrt{0.000212}$$

or,

$$27.7 = \frac{[h(4.0 + h)]^{5/3}}{[4.0 + 2.83h]^{2/3}}$$

- Solving the above equation by iteration: $h = 2.89$ ft (to three significant digits).

(b) Maximum Pressure: Steady-State Flow

- In this case, the maximum pressure will occur at the lowest point in the inverted siphon, at the end of the descending portion of the barrel. This pressure is equal to the change in elevation from the US free water surface to the end of the descending part of the barrel, minus the friction loss. Using Darcy-Weisbach for h_f :

$$h_{\max} = (2.89 + 983.300 - 954.445) - \frac{8f(185)(40.0)^2}{(32.2)\pi^2(2.25)^5}$$

or,

$$h_{\max} = 31.7 - 129f$$

- Note that f is the same as previously calculated (same D , same Q , etc.): $f = 0.0166$. Then,

$$h_{\max} = 31.7 - 129(0.0166) = 29.6 \text{ ft}$$

and,

$$P_{\max} = \frac{29.6 \text{ ft}}{2.31 \text{ ft/psi}} = 12.8 \text{ psi}$$

(c) Maximum Pressure: Zero Flow

- In this case, the maximum pressure will again occur at the lowest point in the inverted siphon. For zero flow, there is zero friction loss.
- Assuming zero depth in the downstream channel:

$$h_{\max} = 975.950 - 954.445 = 21.5 \text{ ft}$$

and,

$$P_{\max} = \frac{21.5 \text{ ft}}{2.31 \text{ ft/psi}} = 9.31 \text{ psi}$$

- Thus, the pressure is very low under both steady and non-flowing conditions, well within the pressure rating for concrete pipe joints.

(e) Hydraulic Seal at Siphon Entrance

- The hydraulic seal is the difference in elevation between the US free water surface and the crown (highest point) of the barrel inlet. In our case, this is approximately equal to:

$$\text{seal} = 2.89 - 2.25 = 0.64 \text{ ft}$$

- According to USBR design criteria, the required hydraulic seal is 1.5 times the difference in velocity heads between the pipe and the open canal:

$$\text{required seal} = 1.5\Delta h_v$$

- Velocity head in the pipe:

$$\left(\frac{V^2}{2g}\right)_{\text{pipe}} = \frac{8(40.0 \text{ cfs})^2}{(32.2)(\pi(2.25)^2)^2} = 1.57 \text{ ft}$$

- Velocity head in the canal:

$$\left(\frac{V^2}{2g}\right)_{\text{canal}} = \frac{(40.0 \text{ cfs})^2}{2(32.2)(2.89(4 + 2.89))^2} = 0.0627 \text{ ft}$$

- Then,

$$\text{required seal} = 1.5(1.57 - 0.0627) = 2.26 \text{ ft}$$

- Thus, the actual hydraulic seal is much less than the required hydraulic seal.