BIE 5300/6300

Fall Semester 2004
Exam #1 – 19 Oct 04

Show your work neatly on this or separate pages. Show units for all calculated values. Make note of any important assumptions.

Name______________________________

1. (20 pts) A Cutthroat flume needs to be installed in an irrigation canal at a location with a maximum flow rate of 50 cfs. The canal is rectangular in cross section, with a base width of 6.0 ft, and a longitudinal bed slope of 0.00065 ft/ft. Normal depth at the maximum flow rate is 2.40 ft.

(a) Select a standard Cutthroat flume size (specify L & W in feet).
(b) Will the canal cross section need to be modified to accommodate the flume? Be specific.
(c) Determine the minimum height (in feet) of the flume floor with respect to the channel bed such that the flume operates under free-flow conditions at Q_{max}.

2. (15 pts) Current metering data is given below for an open channel with a top width of 3.03 m. Complete the calculations to estimate the total flow rate in m^3/s.

distance from edge (m)	depth (m)	depth fraction	velocity (m/s)						
			at point	mean in vertical	mean in section	mean depth (m)	width (m)	area (m^2)	flow rate (m^3/s)
0.05	0.00	n/a	10%	0.019					
					0.105	0.090	0.400	0.036	0.0038
0.45	0.18	0.6	0.190	0.190					
					0.201	0.375	0.800	0.300	0.0602
1.25	0.57	0.2	0.208	0.211					
		0.8	0.214		0.215	0.700	0.500	0.350	0.0751
1.75	0.83	0.2	0.211	0.218					
		0.8	0.225		0.221	0.900	0.250	0.225	
2.00	0.97	0.2	0.218	0.224					
		0.8	0.229		0.220	1.010	0.500	0.505	
2.50	1.05	0.2	0.210	0.216					
		0.8	0.222		0.213	1.050	0.500		
3.00	1.05	0.2	0.208	0.210					
		0.8	0.212						
3.08	1.05	n/a							
						Totals:			

3. **(15 pts)** You need to design a suppressed, rectangular sharp-crested weir for a rectangular channel with a base width of 2.10 m, and a Q_{max} of 4.5 m^3/s. A normal depth of 0.678 m at Q_{max} was measured in the field. Manning roughness is 0.014. Design the weir, specifying crest height, P, provided the conditions are appropriate.

4. **(25 pts)** A BCW, operating under free-flow conditions, has a flow rate of 17.00 m^3/s when h_u (referenced from the sill) is 1.813 m. At this same flow rate, h_d (referenced from the downstream bed elevation) is 2.661 m. The upstream and throat sections are rectangular with a width of 3.75 m. The sill height is $z_u = z_d = 1.15$ m. This BCW has a downstream ramp with a 6:1 slope.

   (a) Determine the upstream specific energy, E_u.
   (b) Determine h_c (referenced from the sill) for this flow rate.
   (c) Estimate the hydraulic head loss in the diverging section, $(h_f)_{ds}$.

5. **(25 pts)** A circular pipe has a circular, sharp-edged orifice plate (D_2 = 5.50 inches), centered in the pipe cross section, which has an inside diameter of D_1 = 7.90 inches. Taps at D_1 (upstream) and ½D_1 (downstream) are connected to a manometer with mercury, from which a head differential of 186 mm is measured. Water temperature is 13°C. Ignoring thermal expansion adjustments:

   (a) Determine the ratio β.
   (b) Determine the kinematic viscosity, ν.
   (c) Determine the discharge coefficient, C_d.
   (d) Determine the upstream Reynolds number, R_e.
   (e) Determine the flow rate, Q.

Solutions:

1. **(20 pts)** A Cutthroat flume needs to be installed in an irrigation canal at a location with a maximum flow rate of 50 cfs. The canal is rectangular in cross section, with a base width of 6.0 ft, and a longitudinal bed slope of 0.00065 ft/ft. Normal depth at the maximum flow rate is 2.40 ft.

(a) Select a standard Cutthroat flume size (specify L & W in feet).

For 50 cfs, select the W = 3 ft, L = 9 ft Cutthroat flume from the table in the lecture notes, with Q_{max} = 56.9 cfs.

(b) Will the canal cross section need to be modified to accommodate the flume? Be specific.

B = W + L/4.5 = 3 + 9/4.5 = 5 ft. Thus, no modifications to the channel section (which is 6-ft wide) except for the inclusion of headwalls (US & DS) to direct all flow through the flume.

(c) Determine the minimum height (in feet) of the flume floor with respect to the channel bed such that the flume operates under free-flow conditions at Q_{max}.

From Cutthroat flume table in lecture notes: S_t = 0.820, n_f = 1.55, and C_f = 3.442 (ft & cfs) for the selected flume size. At 50 cfs:

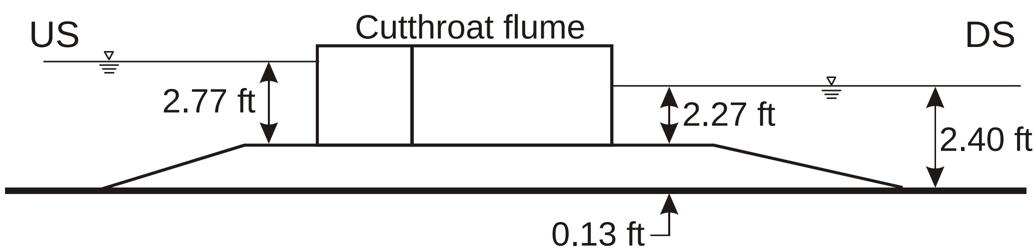
$$h_u = \left(\frac{Q_f}{C_f W}\right)^{1/n_f} = \left(\frac{50.0}{(3.442)(3.0)}\right)^{1/1.55} = 2.77 \text{ ft}$$

Then,

$$\left(h_d\right)_{max} = S_t h_u = (0.820)(2.77) = 2.27 \text{ ft}$$

Finally, the minimum floor height with respect to the channel bed is:

$$\left(h_{floor}\right)_{min} = 2.40 - 2.27 = 0.13 \text{ ft}$$



2. (15 pts) Current metering data is given below for an open channel with a top width of 3.03 m. Complete the calculations to estimate the total flow rate in m^3/s.

Note that there is a vertical wall at the 3.08 distance from the edge (otherwise, the depth would be zero at this location). The ratio x/D is:

$$\frac{x}{D} = \frac{(3.08 - 3.00)}{1.05} = 0.0762$$

Then,

$$\frac{\bar{V}_x}{\bar{V}_D} = \frac{0.65 + 10.52(0.0762)}{1 + 10.676(0.0762) - 0.51431(0.0762)^2} = 0.802$$

and,

$$\bar{V}_w = \frac{0.65(0.210)}{0.802} = 0.170 \text{ m/s}$$

The rest of the calculations are simple and are given in the table below.

distance from edge (m)	depth (m)	depth fraction	velocity (m/s)			mean depth (m)	width (m)	area (m^2)	flow rate (m^3/s)
			at point	mean in vertical	mean in section				
0.05	0.00	n/a	10%	0.019					
					0.105	0.090	0.400	0.036	0.0038
0.45	0.18	0.6	0.190	0.190					
					0.201	0.375	0.800	0.300	0.0602
1.25	0.57	0.2	0.208	0.211					
		0.8	0.214		0.215	0.700	0.500	0.350	0.0751
1.75	0.83	0.2	0.211	0.218					
		0.8	0.225		0.221	0.900	0.250	0.225	0.0497
2.00	0.97	0.2	0.218	0.224					
		0.8	0.229		0.220	1.010	0.500	0.505	0.1110
2.50	1.05	0.2	0.210	0.216					
		0.8	0.222		0.213	1.050	0.500	0.525	0.1118
3.00	1.05	0.2	0.208	0.210					
		0.8	0.212		0.190	1.050	0.080	0.084	0.0160
3.08	1.05	n/a		0.170					
						Totals:	3.030	2.025	0.427

3. (15 pts) You need to design a suppressed, rectangular sharp-crested weir for a rectangular channel with a base width of 2.10 m, and a Q_{max} of 4.5 m^3/s. A normal depth of 0.678 m at Q_{max} was measured in the field. Manning roughness is 0.014. Design the weir, specifying crest height, P, provided the conditions are appropriate.

Check the Froude number in this channel at Q_{max}.

$$F_r^2 = \frac{Q^2 T}{gA^3} = \frac{(4.5)^2(2.10)}{(9.81)\left[(2.10)(0.678)\right]^3} = 1.50$$

Thus, the regime is supercritical, so it is suggested that this is not a good location for a measurement weir.

4. (25 pts) A BCW, operating under free-flow conditions, has a flow rate of 17.00 m^3/s when h_u (referenced from the sill) is 1.813 m. At this same flow rate, h_d (referenced from the downstream bed elevation) is 2.661 m. The upstream and throat sections are rectangular with a width of 3.75 m. The sill height is $z_u = z_d$ = 1.15 m. This BCW has a downstream ramp with a 6:1 slope.

(a) Determine the upstream specific energy, E_u.

$$E_u = h_u + \frac{Q^2}{2gA_u^2} =$$

$$(1.813 + 1.15) + \frac{(17.0)^2}{2(9.81)\left[(1.813+1.15)(3.75)\right]^2} = 3.082 \text{ m}$$

(b) Determine h_c (referenced from the sill) for this flow rate.

For critical flow, the Froude number equals unity. Thus,

$$F_r^2 = \frac{Q^2 T}{gA^3} = \frac{(17.0)^2(3.75)}{9.81\left(3.75 h_c\right)^3} = 1$$

whereby h_c = 1.280 m.

(c) Estimate the hydraulic head loss in the diverging section, $(h_f)_{ds}$.

See the equations in the lecture notes:

$$\xi = \frac{\log_{10}\left[114.6\tan^{-1}(1/6)\right] - 0.165}{1.742} = 0.638$$

The velocity at the critical-flow section is V_c = 17.0/((3.75)(1.280)) = 3.54 m/s. The velocity at the DS section is V_d = 17.0/((3.75)(2.661)) = 1.70 m/s. Then,

$$(h_f)_{ds} = \frac{\xi(V_c - V_d)^2}{2g} = \frac{0.638(3.54 - 1.70)^2}{2(9.81)} \approx 0.11\,\text{m}$$

5. **(25 pts)** A circular pipe has a circular, sharp-edged orifice plate (D_2 = 5.50 inches), centered in the pipe cross section, which has an inside diameter of D_1 = 7.90 inches. Taps at D_1 (upstream) and ½D_1 (downstream) are connected to a manometer with mercury, from which a head differential of 186 mm is measured. Water temperature is 13°C. Ignoring thermal expansion adjustments:

(a) Determine the ratio β.

$$\beta = \frac{D_2}{D_1} = \frac{5.50}{7.90} = 0.696$$

(b) Determine the kinematic viscosity, ν.

$$\nu = \frac{1}{83.9192(13)^2 + 20{,}707.5(13) + 551{,}173} = 1.198(10)^{-6}\ \text{m}^2/\text{s}$$

(c) Determine the discharge coefficient, C_d.

This requires one or two iterations. The diameter of the orifice is: D_2 = 0.3048(5.50/12) = 0.140 m. The cross-sectional area of the orifice opening is:

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi(0.140)^2}{4} = 0.0154\ \text{m}^2$$

Start with C_d = 0.6.

$$Q_1 = 0.6(0.0154)\frac{\sqrt{2g(0.186)(13.6-1)}}{\sqrt{1-(0.696)^4}} = 0.6(0.1194) = 0.0716\ \text{m}^3/\text{s}$$

Then,

$$R_e = \frac{4Q}{\pi D \nu} = \frac{4(0.0716)}{\pi(0.140)(0.000001198)} = 543{,}500$$

and,

$$\begin{aligned} C_d &= 0.5959 + 0.0312(0.696)^{2.1} - 0.184(0.696)^8 \\ &\quad + \frac{0.039(0.696)^4}{1-(0.696)^4} - 0.0158(0.696)^3 + \frac{91.71(0.696)^{2.5}}{(543{,}500)^{0.75}} \\ &= 0.607 + \frac{37.06}{R_e^{0.75}} \\ &= 0.609 \end{aligned}$$

The adjusted flow rate is:

$$Q_2 = 0.609(0.1194) = 0.0727 \text{ m}^3/\text{s}$$

Another iteration gives: R_e = 551,900, and C_d = 0.609, so it is converged.

(d) Determine the upstream Reynolds number, R_e.

R_e = 551,900

(e) Determine the flow rate, Q.

Q = 0.0727 m^3/s